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Lower bounds for open charm and beauty contributions to F_2

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Abstract: On the basis of some peculiar scaling properties of the OPE coefficient functions, we give the lower bounds for $F_2^{c\bar{c}}$ and $F_2^{b\bar{b}}$ independently of the properties of the gluon density function. Predictions for $F_2^{b\bar{b}}$ within reach of HERA are made.

Keywords: LEP HERA and SLC Physics, Heavy Quarks Physics, Deep Inelastic Scattering, QCD.

In deep inelastic ep scattering (DIS) H1 and ZEUS Collaborations can study an inner structure of the proton. The process with open charm or beauty production at HERA is of particular interest, since in these processes there is an additional (to a momentum transfer Q^2) large mass scale related to a mass of a heavy quark. Measurements of the open charm in DIS were mainly done for D and D^*-meson production [1, 2]. The charm contribution to the proton structure, $F_2^{c\bar{c}}$, was estimated. The $b\bar{b}$-cross section was measured in DIS by ZEUS [3] and also in photoproduction by H1 and ZEUS [4]. Recently, the contribution of the beauty to the DIS structure function, $F_2^{b\bar{b}}$, was presented by H1 in Ref. [5] (simultaneously with new data on $F_2^{c\bar{c}}$).

In our previous papers [6, 7], we have made a description of a ratio $F_2^{c\bar{c}}/F_2$ as a function of x for different Q^2 values and have compared our predictions with the HERA data. In the present paper, we calculate lower bonds for both $F_2^{c\bar{c}}/F_2$ and $F_2^{b\bar{b}}$. We will compare our results with the recent data from [2] and [5], and will give predictions concerning behavior of the ratio $F_2^{b\bar{b}}/F_2$ for not yet measured region of Q^2 and x.

Let $\tilde{F}_2^i(Q^2,x)$ be a contribution to DIS structure function F_2 from the quark of type i, with quark charge squared subtracted. Then we can write

$$F_2(Q^2,x) = \sum_i e_i^2\, \tilde{F}_2^i(Q^2,x)\,, \qquad i = u,\, d,\, s,\, c,\, b\,. \tag{1}$$

Let us define (neglecting masses of light quarks, $m_u = m_d = m_s = 0$)

$$\tilde{F}_2^u = \tilde{F}_2^d = \tilde{F}_2^s = \tilde{F}_2^q \tag{2}$$

and introduce a difference between F_2^q and charm (beauty) contribution to the structure function F_2:

$$\Delta\tilde{F}_2^Q = \tilde{F}_2^q - \tilde{F}_2^Q, \qquad Q = c,\, b\,. \tag{3}$$

At small x, the quantity $\tilde{F}_2^i$ can be represented in the following form (neglecting small corrections k^2/Q^2 and m^2/Q^2) [6]:

$$\frac{1}{x}\,\tilde{F}_2^i(Q^2,m_i^2,x) = \int_x^1 \frac{dz}{z}\int_{Q_0^2}^{Q^2(z/x)} \frac{dl^2}{l^2}\; C\!\left(\frac{Q^2}{l^2},\frac{m_i^2}{l^2},\frac{x}{z}\right)\frac{\partial}{\partial \ln l^2}\, g(l^2,z)\,, \tag{4}$$

where $g(k^2,x)$ is a gluon density in momentum fraction x at scale k^2 inside the proton. As for the difference $\Delta\tilde{F}_2^Q(Q^2,m_Q^2,x)$, we have obtained that it tends to a Q^2-independent function at large Q^2 (see [6, 7] for details):

$$\frac{1}{x}\,\Delta\tilde{F}_2^Q(Q^2,m_Q^2,x)\Big|_{Q^2\to\infty} \to \frac{1}{x}\,\Delta\tilde{F}_2^Q(m_Q^2,x) =$$
$$= \int_x^1 \frac{dz}{z}\int_{Q_0^2}^{\infty} \frac{dl^2}{l^2}\;\Delta C\!\left(\frac{m_Q^2}{l^2},\frac{x}{z}\right)\frac{\partial}{\partial \ln l^2}\, g(l^2,z)\,. \tag{5}$$

The coefficient functions $C(u,v,x)$ and $\Delta C(v,x)$ were analytically calculated in the first order in strong coupling constant in refs. [6].[1] The intrinsic charm (beauty) is neglected for $Q^2 \gg Q_0^2 \gg \Lambda_{QCD}^2$ and small x.

[1] If ΔC remains analytical in α_s at $Q^2 \to \infty$, then it seems fairly plausible that its scaling property holds in all orders.

For $0 < x < 0.2$, the coefficient functions in eqs. eq. (4) and eq. (5) obey the inequalities

$$C(u, 0, x) > \Delta C(v, x) > 0\,, \tag{6}$$

from which one can derive the following inequalities for *measurable structure functions* $\mathbf{F_2^{c\bar{c}}}$ *and* $\mathbf{F_2^{b\bar{b}}}$ [6]:

$$\frac{F_2^{c\bar{c}}(Q^2,x)}{F_2(Q^2,x)} > \frac{2}{5}\left[1 - \frac{F_2(m_c^2,x)}{F_2(Q^2,x)}\right], \tag{7}$$

$$\frac{F_2^{b\bar{b}}(Q^2,x)}{F_2(Q^2,x)} > \frac{1}{7}\left[1 - \frac{F_2^{c\bar{c}}(Q^2,x)}{F_2(Q^2,x)} - \frac{F_2(m_b^2,x)}{F_2(Q^2,x)} + \frac{F_2^{c\bar{c}}(m_b^2,x)}{F_2(Q^2,x)}\right]. \tag{8}$$

Following [8], we assumed that $F_2^{c\bar{c}}(m_c^2,x) = F_2^{b\bar{b}}(m_b^2,x) \simeq 0$. We also neglected a small correction $F_2^{b\bar{b}}(Q^2,x)/F_2(Q^2,x)$ in eq. (7), since this ratio is less than 2.5 percent even at high values of Q^2 [5], and, consequently, its contribution to $F_2^{c\bar{c}}(Q^2,x)/F_2(Q^2,x)$ is less than 0.01.

The formulae eq. (4), eq. (5) were derived under assumption that at small x gluons make a leading contribution to a pair production of both charm, bottom and light quarks. Let us estimate a region of x in which our formulae can be applicable. From a NLO DGLAP fit to the proton structure function F_2, we know that $g(Q^2,x)/u_V(Q^2,x) \simeq 6-6.5$ for $x = 0.05$, $Q^2 = 10\,\mathrm{GeV}^2$ (see, for instance [9, figure 3]).[2] Taking into account that the inequalities for the coefficient functions eq. (6) require $x < 0.2$, and that (schematically) $F_2 = C \otimes g\,[x]$ ($\Delta F_2 = \Delta C \otimes g\,[x]$) we get a restriction $x \lesssim 0.01$.

In our previous paper [7], we estimated the lower bound on $F_2^{c\bar{c}}/F_2$ as a function of x for different Q^2 values, and compared our predictions with ZEUS data on a $D^{*\pm}$-production and the charm contribution to F_2 in DIS (taken from the fourth paper in ref. [1]). Recently, the new ZEUS data on the open charm production have appeared [2]. Our predictions are shown in figures 1 and 2 in comparison with all ZEUS data on the $D^{*\pm}$-production. Despite the fact that our curves give only the lower bounds for the ratio $F_2^{c\bar{c}}/F_2$, they are very close to the experimental points and to the ZEUS NLO QCD fit (especially, for $Q^2 \geqslant 18\,\mathrm{GeV}^2$).

Several experimental points in a region $10^{-3} < x < 10^{-2}$ lie somewhat below our curves, although coincide with the lower bounds within 2σ. The effect may be attributed to a contribution from valence quarks which is not negligible at not very small x. In general, the model does not fix exactly "boundary point" value of x, where it is unambiguously valid.

Let us stress that in our approach *no specific expression for the gluon distribution* $\mathbf{g(k^2, x)}$ were assumed. Moreover, in order to calculate the lower bound for $F_2^{c\bar{c}}/F_2$ eq. (7), one needs only the structure function $F_2(Q^2,x)$ which has being measured with a high accuracy. We used a parametrization of H1 from ref. [10].

As we have already mentioned, the beauty production cross section for DIS events has been measured by ZEUS. [3], but only recently H1 could extract $F_2^{b\bar{b}}$ as a function of x for two values of Q^2 [5].

[2] At smaller x (and/or higher Q^2), a gluon dominance is more pronounced. For instance, $g(Q^2,x)$ $/u_V(Q^2,x) \simeq 27-30$ for $x = 0.01$, $Q^2 = 10\,\mathrm{GeV}^2$.

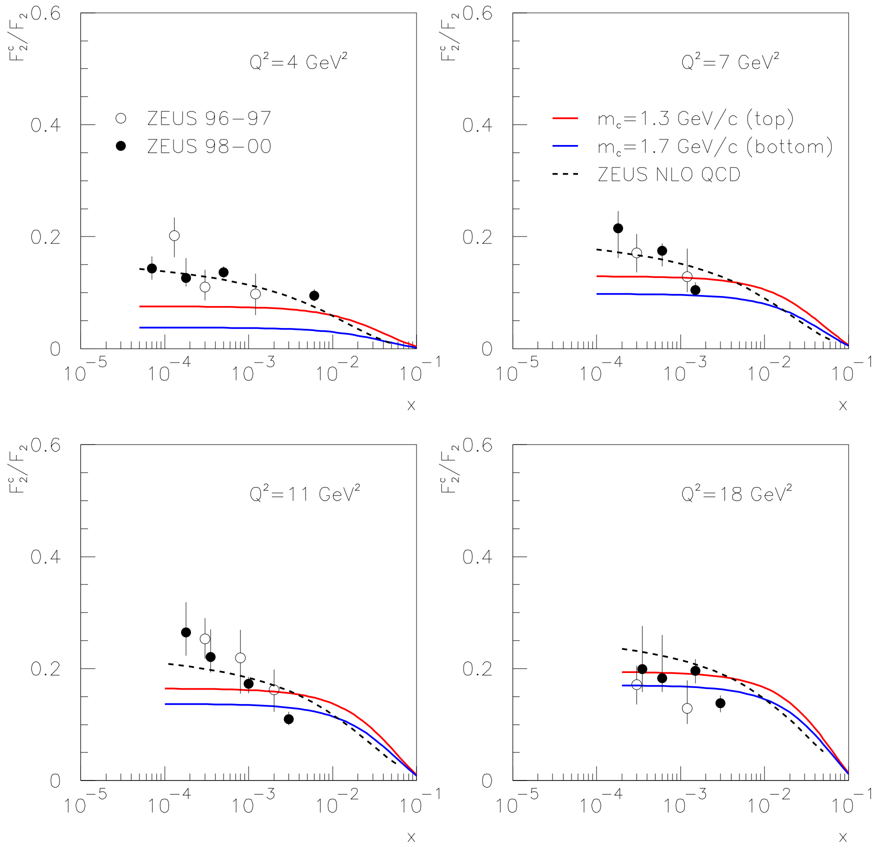

Figure 1: The ratio $F_2^{c\bar{c}}/F_2$ as a function of x for four different Q^2 values. Our predictions for two values of the charm quark mass are shown by solid curves. The ZEUS data on D*-meson production in DIS are taken from [1, 2].

a	b	c	d	e	f	g	h
5.00	1.51	$2.85 \cdot 10^{-4}$	-0.45	- 3.48	4.38	2.01	0.95

Table 1: The result of fitting HERA data on $F_2^{c\bar{c}}$ (h in GeV^2).

If we consider $F_2^{b\bar{b}}(Q^2, x)$ to be a known function of variable Q^2 and x, we may calculate the lower bound for $F_2^{b\bar{b}}$ by using inequality eq. (8). In order to get numerical predictions for $F_2^{b\bar{b}}$, we have fitted the HERA data on open charm production [1, 2] for 4 GeV$^2 \leqslant Q^2 \leqslant$ 500 GeV^2 (71 experimental points with statistical and systematic errors added in quadrature).[3] We used a parametrization similar to that used by H1 for $F_2(Q^2, x)$:

$$F_2^{c\bar{c}}(Q^2, x) = \left[ax^b + cx^d(1 + e\sqrt{x})\left(\ln Q^2 + f\ln^2 Q^2 + \frac{h}{Q^2}\right)\right](1-x)^g\,. \tag{9}$$

The fit gives values of parameters presented in table 1, with a $\chi^2/d.o.f. = 55/63 \simeq 0.87$.

In figure 3, we present both the recent H1 data and a lower bound for $F_2^{b\bar{b}}$ which was calculated with the use of eq. (8) and eq. (9). As in the previous case (description of the ratio $F_2^{c\bar{c}}/F_2$), our predictions are very closed to the experimental points.

[3]The recent H1 data on $F_2^{c\bar{c}}$ [5] (4 points) were not included in the fit.

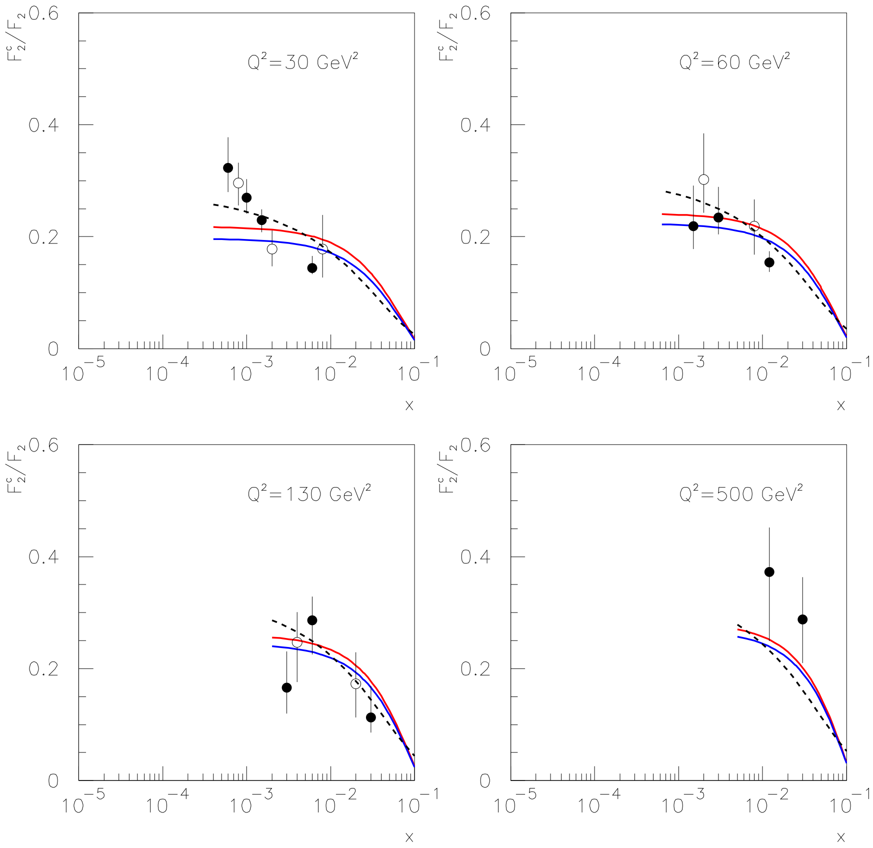

Figure 2: The same as in figure 1 but for higher values of Q^2.

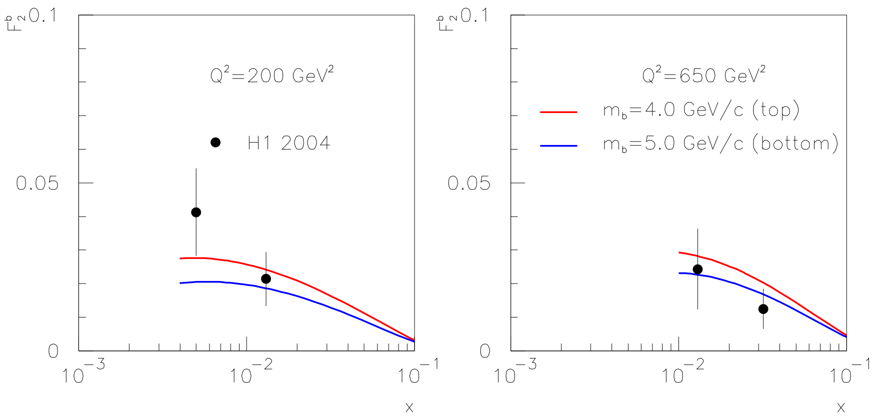

Figure 3: The beauty contribution to the structure function, $F_2^{b\bar{b}}$, as a function of x for two Q^2 values. Our predictions for two values of the bottom quark mass are shown by solid curves. The H1 data on open bottom production are taken from [5].

At small x and large Q^2, theoretical predictions for the structure functions $F_2^{c\bar{c}}$ and $F_2^{b\bar{b}}$ depend crucially on the form of the gluon distribution $g(Q^2, x)$. There are several fits of the parton distribution functions [11 − 13]. To determine quark and gluon distributions, it is necessary to make a global analysis of a wide range of DIS and other hard processes.

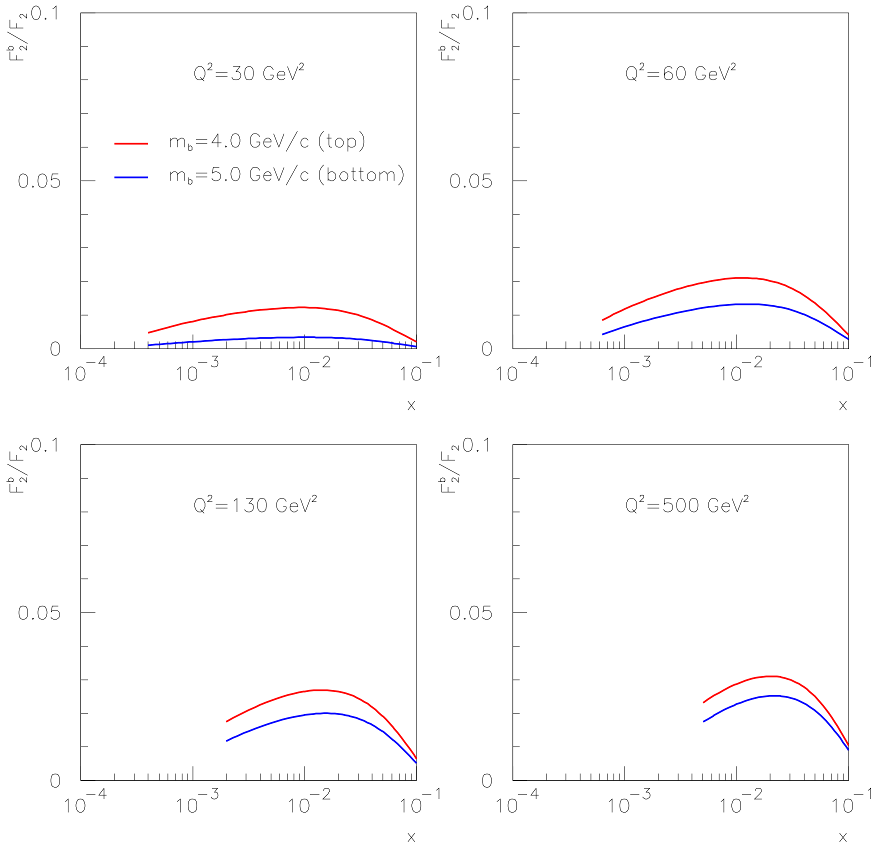

Figure 4: Our prediction for the ratio $F_2^{b\bar{b}}/F_2$. The values of Q^2 and range of x are taken the same as those chosen by ZEUS in open charm measurements (see figure 2).

Nevertheless, this procedure has a lot of experimental an theoretical uncertainties (see [14] and references therein). In the present paper, we have given the lower bounds on $F_2^{c\bar{c}}$ and $F_2^{b\bar{b}}$, which, however, *do not depend on the form of* $\mathbf{g(Q^2, x)}$ *and on approximation used for its calculation.* The only things we need are the properties of the coefficient functions[4] (see eq. (6)) and the rise of the gluon distribution in variable $\ln Q^2$ at $x \lesssim 0.01$, $\partial g(Q^2, x)/\partial \ln Q^2 > 0$.

As we have already seen, the available experimental data on open charm and beauty productions seem to saturate the lower bounds on $F_2^{c\bar{c}}(Q^2, x)$ and $F_2^{b\bar{b}}(Q^2, x)$, at least at $Q^2 \geqslant 18\,\mathrm{GeV}^2$. That is why, we expect that our predictions for $F_2^{b\bar{b}}/F_2$ (see figure 4) will be useful for future measurements of the structure function $F_2^{b\bar{b}}$ at HERA.

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[4]Let us remind that the coefficient functions C, ΔC in Eqs. eq. (4), eq. (5) were calculated in order $\mathrm{O}(\alpha_s)$, with accounting for both a gluon virtuality and a quark mass inside a quark loop [6].

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